# New Branches of Solutions – From Sphalerons to Black Holes

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Yves Brihaye Fest, Mons, June 2-3, 2022

# Long history of collaboration

# 1986

# Long history of collaboration

# Long history of collaboration

# Long history of collaboration

# Outline

1. From Flat to Curved Space
   - Electroweak Sphalerons
   - Dyons and Hairy Black Holes
2. Neutron Stars & Black Holes
   - Matter Induced Scalarization
   - Curvature Induced Scalarization
   - Spin Induced Scalarization
3. Conclusions

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# Golden age of collaboration: 1987 – 1999

# New Sphalerons in Weinberg-Salam Theory

Volume 216, number 3,4 PHYSICS LETTERS B 12 January 1989

## NEW SPHALERONS IN THE WEINBERG–SALAM THEORY

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We discovered new sphalerons in the Weinberg–Salam theory, in the limit of vanishing mixing angle. Dependent on the mass of the Higgs field, a whole set of saddle-point solutions exists. The Dashen–Hasslacher–Neveu sphaleron forms the basic branch of solutions from which, at critical values of the Higgs mass, new branches of solutions systematically emerge.

# New Sphalerons in Weinberg-Salam Theory

Lagrangian of $SU(2)$-Higgs model ($\theta_w = 0$)

$$L = -\frac{1}{2g^2}\mathrm{tr}(F^{\mu\nu}F_{\mu\nu}) + D_\mu\phi D^\mu\phi^\dagger - \lambda(\phi\phi^\dagger - \frac{v^2}{2})^2$$

gauge field tensor

$$F_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu - i[V_\mu, V_\nu]$$

covariant derivative

$$D_\mu\phi = (\partial_\mu - iV_\mu)\phi$$

Higgs field vacuum expectation value

$$\langle\phi\rangle = \frac{v}{\sqrt{2}}\begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

masses of gauge and Higgs bosons

$$M_W = \frac{vg}{2} \quad , \quad M_H = v\sqrt{2\lambda}$$

# New Sphalerons in Weinberg-Salam Theory

static spherically symmetric solutions

Higgs field

$$\phi = \frac{v}{\sqrt{2}} L(r) \exp\left[i\vec{\tau}\cdot\hat{r}\, F(r)\right] \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

gauge field

$$\begin{aligned} V_\mu &= \frac{1}{2} g \tau_a V_\mu^a \\ V_0^a &= 0 \\ V_i^a &= \frac{G(r)}{gr}\epsilon_{aib}\hat{r}_b + \frac{H(r)}{gr}\left(\delta_{ai} - \hat{r}_a\hat{r}_i\right) + \frac{K(r)}{gr}\hat{r}_a\hat{r}_i \end{aligned}$$

gauge choice $F(r) = 0$

sphaleron

$$H(r) = K(r) = 0$$

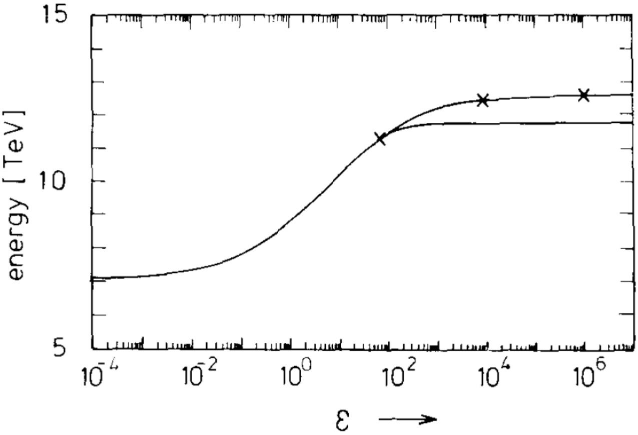
bisphaleron

$$H(r) \neq 0 \ , \quad K(r) \neq 0$$

new branches of solutions

# New Sphalerons in Weinberg-Salam Theory

bisphalerons branch off the sphaleron




energy vs mass ratio $\varepsilon = \frac{1}{2}\left(\frac{M_H}{M_W}\right)^2$

# New Sphalerons in Weinberg-Salam Theory

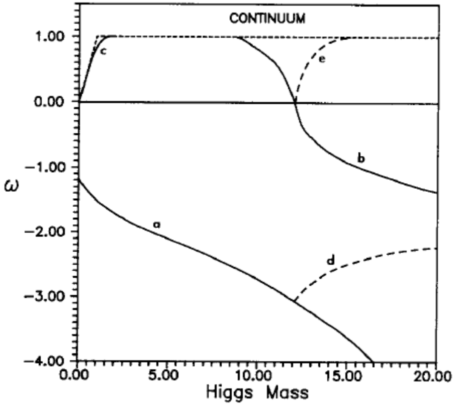
perturbing the sphaleron

small fluctuations around background functions: normal modes

set of coupled equations: eigenvalue problem

# New Sphalerons in Weinberg-Salam Theory

normal modes of sphaleron and bisphaleron




energy vs mass ratio $M_H/M_W$

solid: sphaleron normal modes, dashed: bisphaleron normal modes

# Outline



# New collaborator Betti Hartmann

# Gravitating Dyons

26 November 1998

PHYSICS LETTERS B

ELSEVIER

Physics Letters B 441 (1998) 77–82

## Gravitating dyons and dyonic black holes

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Editor: P.V. Landshoff

**Abstract**

We study static spherically symmetric gravitating dyon solutions and dyonic black holes in Einstein-Yang-Mills-Higgs theory. The gravitating dyon solutions share many features with the gravitating monopole solutions. In particular, gravitating dyon solutions and dyonic black holes exist only up to a maximal coupling constant, and beside the fundamental dyon solutions there are excited dyon solutions. 

# Gravitating Dyons

SU(2) Einstein-Yang-Mills-Higgs action: Higgs triplet

$$S = \int d^4x\sqrt{-g}\left\{\frac{1}{16\pi G}R - \frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} - \frac{1}{2}D_\mu\phi^a D^\mu\phi^a - \frac{1}{4}\lambda(\phi^a\phi^a - H_0^2)^2\right\}$$

field strength tensor

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + e\epsilon^{abc}A^b_\mu A^c_\nu$$

covariant derivative

$$D_\mu\phi^a = \partial_\mu\phi^a + e\epsilon^{abc}A^b_\mu\phi^c$$

gauge coupling $e$, Higgs coupling $\lambda$, and Higgs field expectation value $H_0$

gravitating magnetic monopoles and black holes with monopole hair were known

how about gravitating dyons and black holes with dyonic hair?

# Gravitating Dyons

static spherically symmetric metric in Schwarzschild-like coordinates

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = -A^2 N dt^2 + N^{-1} dr^2 + r^2 (d\theta^2 + \sin^2\theta d\phi^2)$$

$$N = 1 - \frac{2m}{r}$$

spherically symmetric ansatz for the gauge and Higgs field

$$\vec{A}_t = \vec{e}_r J(r) H_0$$

$$\vec{A}_r = 0 \ , \quad \vec{A}_\theta = \vec{e}_\phi \frac{1 - K(r)}{g} \ , \quad \vec{A}_\phi = -\vec{e}_\theta \frac{1 - K(r)}{g} \sin\theta$$

$$\vec{\phi} = \vec{e}_r H(r) H_0$$

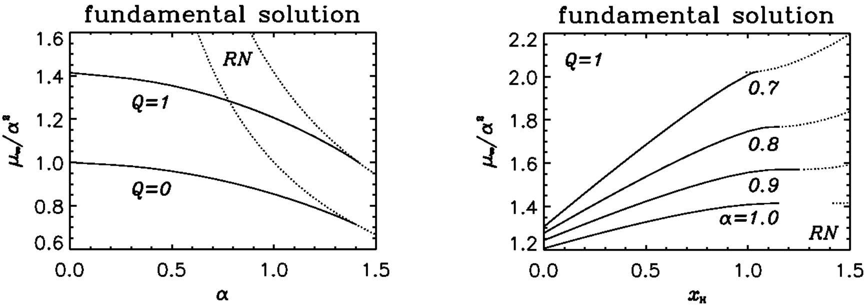
dimensionless coordinate $x$

$$x = e H_0 r$$

coupling constants $$\alpha^2 = 4\pi G H_0^2 \ , \quad \beta^2 = \frac{\lambda}{g^2}$$

# Gravitating Dyons

gravitating dyons and hairy dyonic black holes




mass vs coupling $\alpha$ mass vs horizon radius $x_H$

bifurcations (?) from Reissner-Nordström black hole

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# Scalar-Tensor Theories

action: Einstein frame

$$S = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\,[\mathcal{R} - 2g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - 4V(\varphi)] + S_m[\Psi_m; \mathrm{A}^2(\varphi)g_{\mu\nu}]$$

metric in Einstein frame: $g_{\mu\nu}$

scalar field in Einstein frame: $\varphi$

non-minimal coupling functon to matter: $A(\varphi)$

pressure and density in Einstein frame: $p$, $\rho$

$$p = A^4\tilde{p}\ , \quad \rho = A^4\tilde{\rho}$$

# Scalar-Tensor Theories: Spontaneous Scalarization

VOLUME 70, NUMBER 15 PHYSICAL REVIEW LETTERS 12 APRIL 1993

**Nonperturbative Strong-Field Effects in Tensor-Scalar Theories of Gravitation**

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Gilles Esposito-Farèse
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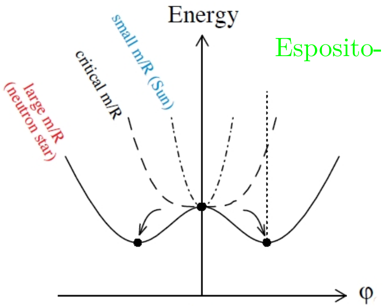



Esposito-Farese (lecture)

matter induced **"spontaneous scalarization"**

# Scalar-Tensor Theories: Spontaneous Scalarization

Damour and Esposito-Farèse (1993)

Einstein frame: field equations

$$R_{\mu\nu} = 2\partial_\mu\varphi\partial_\nu\varphi + 8\pi G\left(T_{\mu\nu} - \frac{1}{2}Tg_{\mu\nu}\right)$$

$$\nabla_\mu\nabla^\mu\varphi + 4\pi G\alpha(\varphi)T = \nabla_\mu\nabla^\mu\varphi - m_{\rm eff}^2\varphi = 0$$

Klein-Gordon equation with effective mass term $m_{\rm eff}^2$
coupling function

$$A = \exp\left(\frac{1}{2}\beta_0\varphi^2\right), \quad \alpha(\varphi) = \frac{\partial \ln A(\varphi)}{\partial\varphi} = \beta_0\varphi, \quad \beta_0 < 0$$

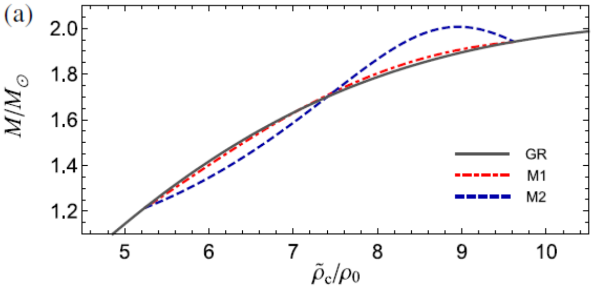
GR solutions remain solutions: $\varphi = 0$, $\alpha(\varphi) = 0$

matter induced tachyonic instability: $m_{\rm eff}^2 < 0$

$$m_{\rm eff}^2 = -4\pi G\frac{\alpha(\varphi)}{\varphi}T = -4\pi G\beta_0 T < 0\ , \quad \text{if} \ \ T < 0$$

# Scalar-Tensor Theories: Spontaneous Scalarization

Mendes et al., 1802.07847




mass – central density ($\beta = -5$)

$$\text{Model2}(M2): A(\varphi) \quad = \quad e^{\beta\varphi^2/2}$$
$$\text{Model1}(M1): A(\varphi) \quad = \quad [\cosh(\sqrt{3}\beta\varphi)]^{1/(3\beta)}$$

GR: zero mode

$\omega = 0$

M2: zero mode

$\omega = 0$

# Scalar-Tensor Theories: Spontaneous Scalarization

Mendes et al., 1802.07847

radial modes

$$\delta\phi(t,r) = \delta\phi(r)e^{i\omega t}, \ldots$$

Eigenvalue $\omega$

$$\omega = \omega_R + i\omega_I$$

$$\delta\phi \sim e^{i\omega_R t}e^{-\omega_I t}, \ldots$$

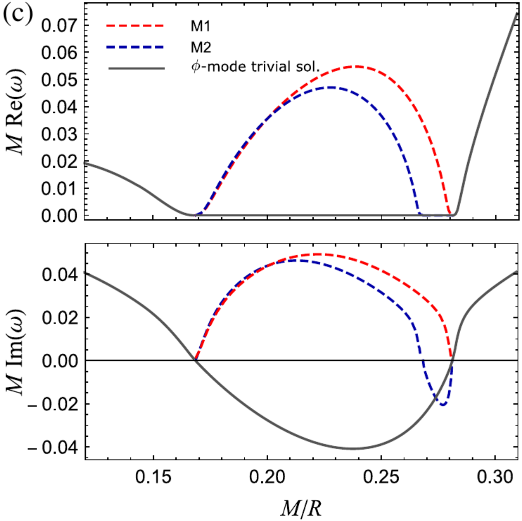
scalarization:

GR: zero mode

$$\omega = 0$$

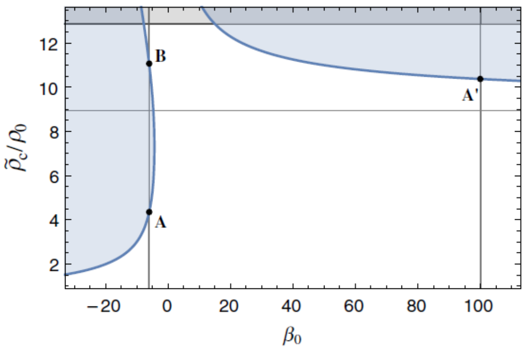
GR: unstable mode

$$\omega_I < 0$$



# Scalar-Tensor Theories: Spontaneous Scalarization

Mendes et al. 1604.04175




1. spontaneous scalarization: $\beta_0 < 0$, $T < 0 \Longrightarrow \beta_0 T > 0$
2. spontaneous scalarization: $\beta_0 > 0$, $T > 0 \Longrightarrow \beta_0 T > 0$

# Outline



# Einstein-scalar-Gauss-Bonnet Theories

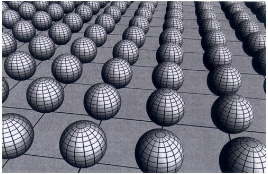
EsGB action

$$S = \frac{1}{16\pi} \int d^4x \sqrt{-g} \left[ R - \frac{1}{2} (\partial_\mu \varphi)^2 + f(\varphi) R^2_{\rm GB} \right]$$

Gauss-Bonnet term: quadratic in the curvature

$$R^2_{\rm GB} = R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} - 4 R_{\mu\nu} R^{\mu\nu} + R^2$$

coupling function $f(\varphi)$



In 4 spacetime dimensions the coupling to the scalar is needed.

The resulting set of equations of motion are of second order (Horndeski).

# Static EsGB black holes

Doneva et al. arXiv:1711.01187, Silva et al. arXiv:1711.02080, Antoniou et al. arXiv:1711.03390

curvature induced scalarized black holes

action

$$S = \frac{1}{16\pi} \int d^4x \sqrt{-g} \Big[ R - 2\nabla_\mu \varphi \nabla^\mu \varphi - V(\varphi) + f(\varphi) \mathcal{R}^2_{GB} \Big]$$

Einstein equations

$$G_{\mu\nu} = T_{\mu\nu}$$

scalar equation

$$\nabla_\mu \nabla^\mu \varphi + \frac{df}{d\varphi} R^2_{\rm GB} = 0$$

GR solutions remain solutions: $\varphi = 0, \ \dfrac{df(\varphi)}{d\varphi} = 0$

tachyonic instability

# Static EsGB black holes

Doneva et al. arXiv:1711.01187, Silva et al. arXiv:1711.02080, Antoniou et al. arXiv:1711.03390

curvature induced scalarized black holes

scalar equation

$$\nabla_\mu \nabla^\mu \varphi + \frac{df}{d\varphi} R^2_{\rm GB} = 0$$

simplest choice

$$f(\varphi) = \eta \frac{\varphi^2}{2} \ , \quad \frac{df}{d\varphi} = \eta \varphi$$

Gauss-Bonnet: Schwarzschild

$$R^2_{\rm GB} = \frac{48 M^2}{r^6}$$

effective mass

$$m^2 = -\eta R^2_{\rm GB} < 0 \ , \quad \text{if } \eta > 0$$

# Static EsGB black holes

Doneva et al. arXiv:1711.01187

coupling function

$$f(\varphi) = \frac{\lambda^2}{12}\left(1 - e^{-6\varphi^2}\right)$$

scalar perturbation equation

$$\nabla_\mu \nabla^\mu \delta\varphi + \frac{\lambda^2}{4} R^2_{\rm GB} \delta\varphi = 0$$

$$\delta\varphi = \frac{u(r)}{r} e^{-i\omega t} Y_{lm}(\theta\phi)$$

$$\frac{d^2u}{dr_*^2} = \left(\omega^2 - U(r)\right) u = 0$$

$$U = \left(1 - \frac{2M}{r}\right)\left(\frac{2M}{r^3} + \frac{l(l+1)}{r^2} - \lambda^2 \frac{12M^2}{r^6}\right)$$

Schwarzschild is unstable:

$$M^2 < 0.34\lambda^2$$

# Static EsGB black holes

Doneva et al. arXiv:1711.01187

coupling function

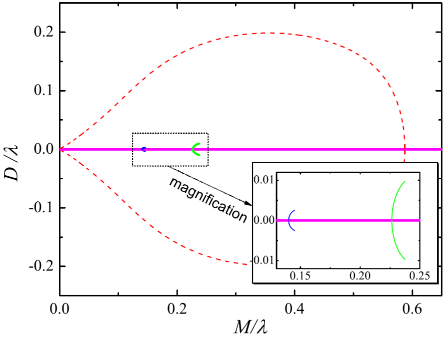
$$f(\varphi) = \frac{\lambda^2}{12}\left(1 - e^{-6\varphi^2}\right)$$

small $\varphi$

$$f(\varphi) = \frac{\lambda^2}{2}\varphi^2$$

sequence of radial excitations

onset $n = 0$: $M/\lambda = 0.587$
onset $n = 1$: $M/\lambda = 0.226$
onset $n = 2$: $M/\lambda = 0.140$




Schwarzschild fat purple
scalarized $n = 0$ red
scalarized $n = 1$ green
scalarized $n = 2$ blue

# Static EsGB black holes

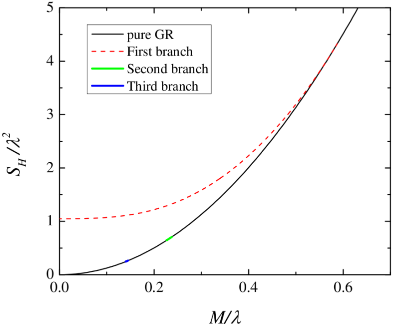
Doneva et al. arXiv:1711.01187

coupling function

$$f(\varphi) = \frac{\lambda^2}{12}\left(1 - e^{-6\varphi^2}\right)$$

small $\varphi$

$$f(\varphi) = \frac{\lambda^2}{2}\varphi^2$$




sequence of radial excitations

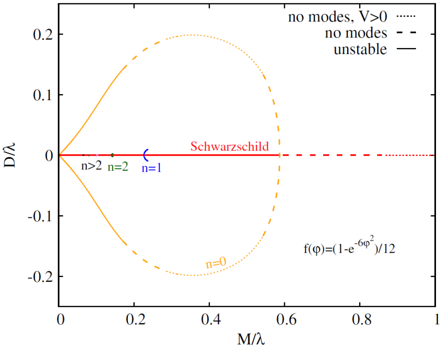
onset $n = 0$: $M/\lambda = 0.587$
onset $n = 1$: $M/\lambda = 0.226$
onset $n = 2$: $M/\lambda = 0.140$

Schwarzschild fat black

scalarized $n = 0$ red
scalarized $n = 1$ green
scalarized $n = 2$ blue

# Static EsGB black holes

Blazquez-Salcedo et al. arXiv:1805.05755




solutions

Schwarzschild red

scalarized $n = 0$ orange

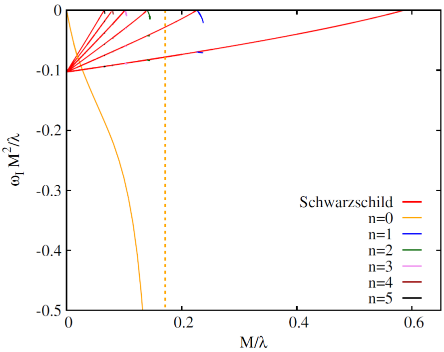
scalarized $n > 0$ ...

unstable radial modes

Schwarzschild red

scalarized $n = 0$ orange

scalarized $n > 0$ ...



# Static EsGB black holes

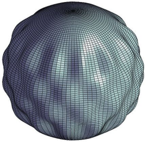
perturbation theory: damped oscillations (QNMs)

metric

$$g_{\mu\nu} = g^{(0)}_{\mu\nu}(r) + \epsilon h_{\mu\nu}(t, r, \theta, \varphi)$$

scalar

$$\phi = \phi_0(r) + \epsilon \delta\phi(t, r, \theta, \varphi)$$



polar modes: even-parity perturbations

axial modes: odd-parity perturbations (pure space-time modes)

master equation: Schrödinger-like equation

eigenvalue $\omega$

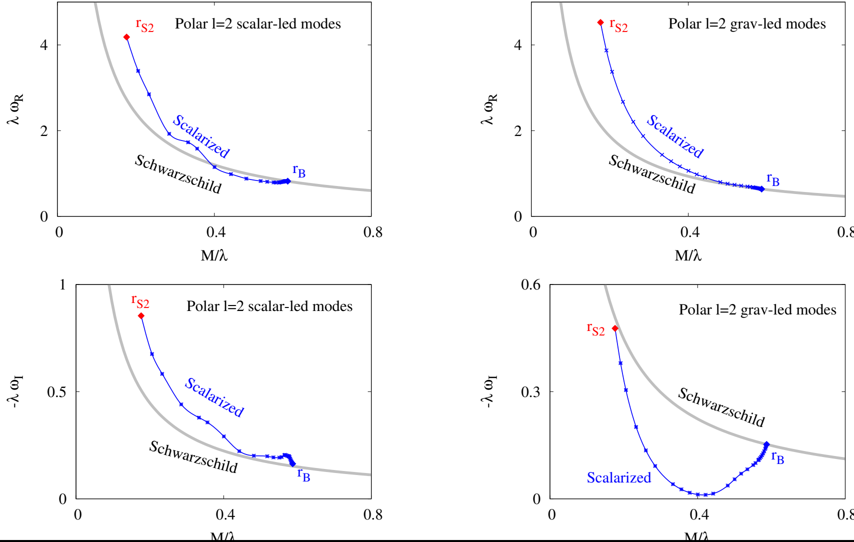
$$\omega = \omega_R + i\omega_I$$

frequency: $\omega_R$

decay time: $\tau = -1/\omega_I$

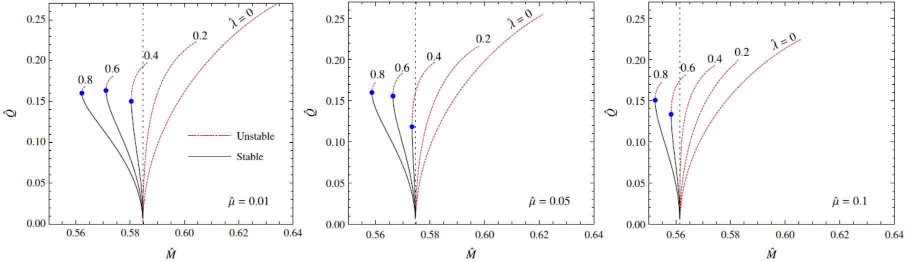
# Static EsGB black holes

Blazquez-Salcedo et al. arXiv:2006.06006



# Static EsGB black holes

Macedo et al. arXiv:1903.06784




quadratic coupling function

scalar field potential

$$V(\varphi) = \frac{1}{2}\mu^2\varphi^2 + \frac{1}{8}\lambda\varphi^4$$

radial stability: small mass, large self-interaction

# Rotating EsGB black holes

Cunha et al. arXiv:1904.09997

curvature induced scalarized black holes

scalar equation

$$\nabla_\mu \nabla^\mu \varphi + \frac{df}{d\varphi} R^2_{\rm GB} = 0$$

Gauss-Bonnet: Kerr

$$R^2_{\rm GB} = \frac{48M^2}{(r^2+\chi^2)^6}\left(r^6 - 15r^4\chi^2 + 15r^2\chi^4 - \chi^6\right) \ , \quad \chi = a\cos\theta$$

effective mass

$$m^2 = -\eta R^2_{\rm GB} < 0$$

rotation suppresses scalarization

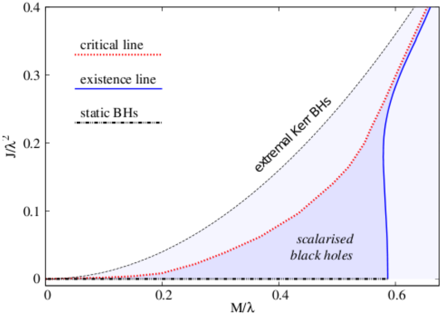
$$\eta > 0$$

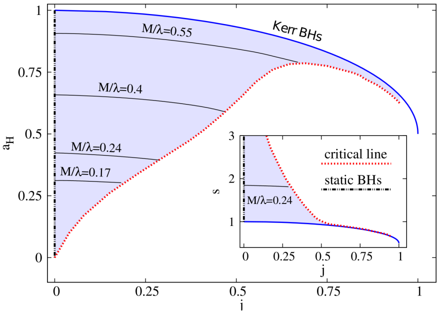
# Rotating EsGB black holes

Cunha et al. arXiv:1904.09997

coupling function

$$f(\varphi) = \frac{\lambda^2}{12}\left(1 - e^{-6\varphi^2}\right) \ , \qquad V(\varphi) = 0$$



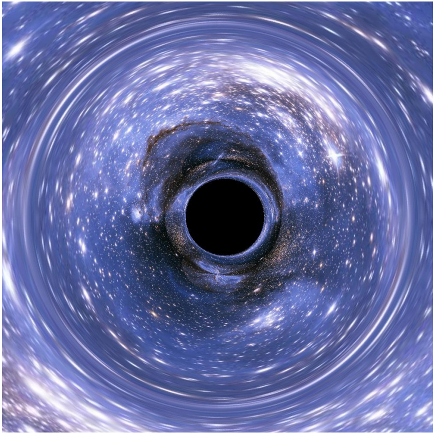




angular momentum vs mass

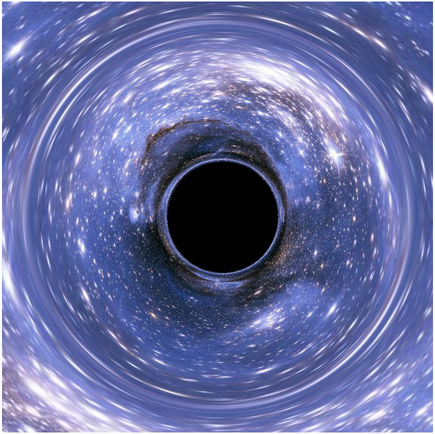
area/entropy vs angular momentum

# Rotating EsGB black holes

Cunha et al. arXiv:1904.09997



EsGB



Kerr

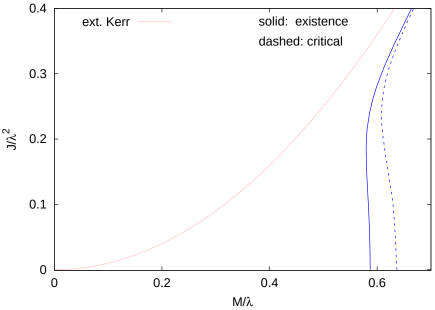
$$M/\lambda = 0.237 (j = 0.24)$$

# Rotating EsGB black holes

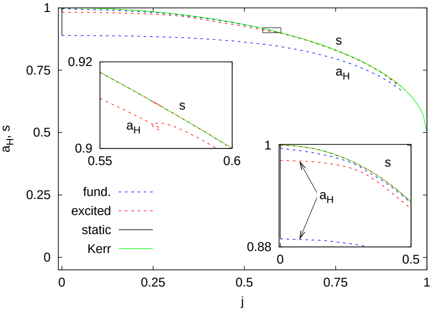
Collodel et al. arXiv:1912.05382

coupling function

$$f(\varphi) = \frac{\lambda^2}{8}\varphi^2 \;, \qquad V(\varphi) = 0$$




angular momentum vs mass




area/entropy vs angular momentum

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# Rotating EsGB black holes

Dima et al. arXiv:2006.03095

curvature induced scalarized black holes

scalar equation

$$\nabla_\mu \nabla^\mu \varphi + \frac{df}{d\varphi} R^2_{\mathrm{GB}} = 0$$

Gauss-Bonnet: Kerr

$$R^2_{\mathrm{GB}} = \frac{48M^2}{(r^2+\chi^2)^6}\left(r^6 - 15r^4\chi^2 + 15r^2\chi^4 - \chi^6\right) \ , \quad \chi = a\cos\theta$$

effective mass

$$m^2 = -\eta R^2_{\mathrm{GB}} < 0$$

sufficiently fast rotation induces scalarization

$$\eta < 0$$

# Rotating EsGB black holes

Dima et al. arXiv:2006.03095

$|\phi| \sim \exp(t/\tau)$

coupling function

$$f(\varphi) = -\eta\,\varphi^2$$

$$V(\varphi) = 0$$

onset of scalarization

even scalar field

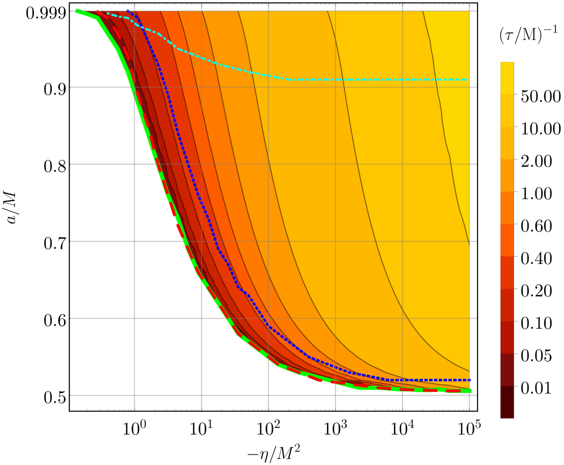
$$\varphi(\pi - \theta) = +\varphi(\theta)$$

odd scalar field

$$\varphi(\pi - \theta) = -\varphi(\theta)$$

range

$$0.5 \le j \le 1$$

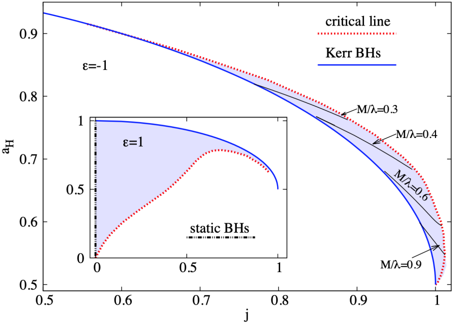


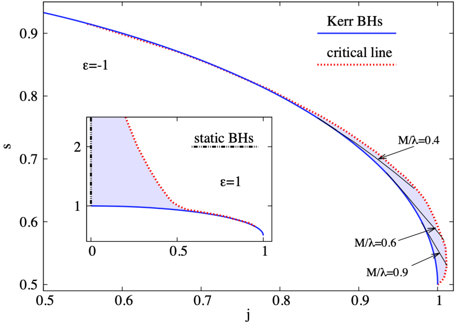
# Rotating EsGB black holes

Herdeiro et al. arXiv:2009.03904

coupling function

$$f(\varphi) = \frac{\lambda^2}{12}\left(1 - e^{-6\varphi^2}\right) \ , \qquad V(\varphi) = 0$$




area vs angular momentum

entropy vs angular momentum

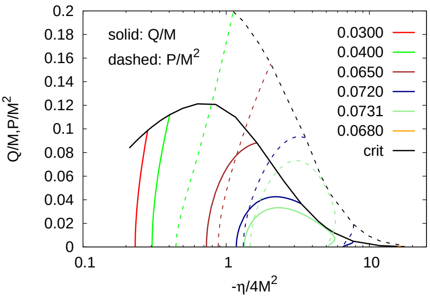
even scalar field

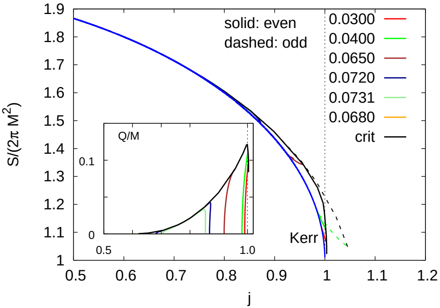
# Rotating EsGB black holes

Berti et al. arXiv:2009.03905

coupling function

$$f(\varphi) = \frac{\eta}{8}\varphi^2 \ , \quad \eta < 0 \ , \qquad V(\varphi) = 0$$




charge/dipole moment vs coupling

entropy vs angular momentum

even ($Q$) and odd ($P$) scalar field

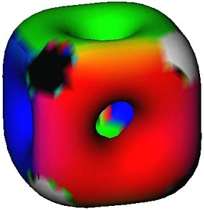
# Outline



# Conclusions

## From Flat to Curved Space

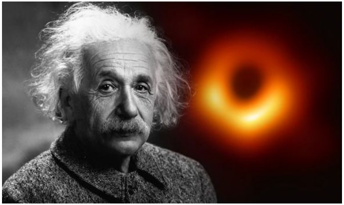
flat space



- sphalerons
- monopoles & dyons
- skyrmions
- ...

curved space

- gravitating sphalerons
- gravitating solitons
- black holes
- ...



# THANKS